Some exact solutions of F(R) gravity with charged (a)dS black hole interpretation

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Abstract

In this paper we obtain topological static solutions of some kind of pure $F(R)$ gravity. The present solutions are two kind: first type is uncharged solution which corresponds with the topological (a)dS Schwarzschild solution and second type has electric charge and is equivalent to the Einstein-Λ-conformally invariant Maxwell solution. In other word, starting from pure gravity leads to (charged) Einstein-Λ solutions which we interpreted them as (charged) (a)dS black hole solutions of pure $F(R)$ gravity. Calculating the Ricci and Kreschmann scalars show that there is a curvature singularity at $r = 0$. We should note that the Kreschmann scalar of charged solutions goes to infinity as $r \to 0$, but with a rate slower than that of uncharged solutions.

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I. INTRODUCTION

One of the main topic in cosmology is the complicated fact that our Universe have an accelerated expansion [1]. In order to interpret this expansion, some various candidates have been proposed by many authors, in particular, cosmological constant idea, dark energy models and modified gravities. Also, transition from linear Einstein-Hilbert action to nonlinear modification of it, is expected when string/M-theory corrections are considered [2].

Amongst the nonlinear modifications of Einstein gravity, the so-called $F(R)$ gravity [3–7], whose action is a nonlinear function of the curvature scalar R, is completely special. It seems to provide an interpretation to dark energy, the hierarchy problem [8], the four cosmological phases [9], the power law early-time inflation [10, 11], late-time cosmic accelerated expansion [11–14], singularity problem arising in the strong gravity regime [15–19], rotation curves of spiral galaxies [20] and detection of gravitational waves [21].

In additions, we can give other motivations to consider $F(R)$ gravity. First of all, in spite of $F(R)$ theory is the simplest modification of the gravitational interaction to higher order known so far, $F(R)$ action is sufficiently general to encapsulate some of the basic characteristics of higher order gravity. Second, it is believed that there are sufficient theoretical predictions to claim that $F(R)$ gravity can be compatible with Newtonian and post-Newtonian prescriptions [22, 23]. Third, there are serious reasons to believe that $F(R)$ theories are unique among higher order gravity theories, in the sense that they seem to be the only ones which can avoid the long known and fatal Ostrogradski instability [24]. Fourth, $F(R)$ theories have no ghosts ($\frac{dF}{dR} > 0$), and the stability condition $\frac{d^2F}{dR^2} \geq 0$ of essentially amounts to guarantee that the scalaron is not a tachyon [25]. All these properties can be easily obtained either directly, or using conformal equivalence of field equations of $F(R)$ theories to those of the Einstein gravity interacting with a minimally coupled scalar field with some potential $V(\phi)$ which form is uniquely determined by $F(R)$ in all points where $\frac{dF}{dR} \neq 0$ [26].

Some of these reasons are sufficient to state why there is much activity in the study of different versions of modified $F(R)$ theories with applications to gravity, cosmology and astrophysics. Unlike general relativity, which involves metric derivatives no higher than second order, $F(R)$ gravity involves also third and fourth order derivatives, which caused complications in the calculation. Some of the exact solutions in $F(R)$ gravity have been

studied in Ref. [27]. Most of this solutions are Schwarzschild with or without of cosmological constant. In Refs. [28–30], it has shown that one may obtain Reissner-Nordström solutions in $F(R)$-Maxwell gravity.

When we follow the Einstein's thought and desires, we may find that one of the weighty dream of Einstein was creating a geometrical unified theory of physics. However the Einstein dream is still very much alive, but till now, all attempts of geometrizing physics are unfeasible. In what follows, we consider one of the modified theory of gravity ($F(R)$ gravity) instead of Einstein general relativity, and show that some solutions of $F(R)$ gravity (pure geometry) are equivalent to the Einstein-nonlinear Maxwell gravity in the present of cosmological constant. It is notable that (a)dS solutions of $F(R) = R + f(R)$ gravity has been investigated before [9, 12], but there is not any charged (a)dS solution of pure $F(R) = R + f(R)$ gravity, yet.

Finally, we should notice as an example of charged solutions in pure $F(R)$ gravity, we refer to the particular form $F(R) = R^N$ [6, 7] (it is not in the form of $F(R) = R + f(R)$), which attains an electromagnetic - like curvature source, so that $N \neq 1$ can be interpreted as an electric charge "without charge". In other word, $F(R) = R^N$ behaves geometrically as if we have $F(R) = R+$ (electrostatic field). Remarkably, $N - 1$ plays the role of "charge" so that the geometry $F(R) = R^N$ becomes equivalent to the Reissner-Nordstrom geometry in a spherically symmetry spacetime. Beside the case of $F(R) = R+$ (electrostatic field), and more aptly, cases such as $F(R) = R+$ (non-minimal scalar field) cases also have been investigated [31].

II. BASIC FIELD EQUATIONS AND METRIC ANSATZ

The action of d-dimensional $F(R)$ gravity, in the presence of matter field has the form of

$$\mathcal{I}_G = -\frac{1}{16\pi} \int d^d x \sqrt{-g} \left[F(R) + \mathcal{L}_{matt} \right], \tag{1}$$

where $F(R)$ is an arbitrary function of Ricci scalar R, and $\mathcal{L}_{matt}$ is the Lagrangian of matter fields. Variation with respect to metric $g_{\mu\nu}$, leads to the following field equations

$$R_{\mu\nu} F_R - \nabla_\mu \nabla_\nu F_R + \left(\Box F_R - \frac{1}{2} F(R) \right) g_{\mu\nu} = \mathrm{T}_{\mu\nu}^{matt}, \tag{2}$$

where $R_{\mu\nu}$ is the Ricci tensor, $F_R \equiv dF(R)/dR$ and $\mathrm{T}_{\mu\nu}^{matt}$ is the standard matter stress-energy tensor in which derived from the matter Lagrangian $\mathcal{L}_{matt}$ in the action (1). The

trace of Eq. (2) reduces to

$$[R + (d-1)\Box] F_R - \frac{d}{2} F(R) = \mathrm{T}, \tag{3}$$

where T is the trace of matter stress-energy tensor. It is notable that Eq. (3) leads to $R = 0$, for sourceless Einstein gravity ($\mathrm{T} = 0$, $F(R) = R$ and $F_R = 1$).

Here, we want to obtain the static solutions of Eq. (2) with positive, negative and zero curvature horizons. For this purpose, we assume that the metric has the following form

$$ds^2 = -g(r)dt^2 + \frac{dr^2}{g(r)} + r^2 d\Omega_k^2 \tag{4}$$

where

$$d\Omega_k^2 = \begin{cases} d\theta_1^2 + \sum\limits_{i=2}^{d-2} \prod\limits_{j=1}^{i-1} \sin^2\theta_j d\theta_i^2 & k=1 \\ d\theta_1^2 + \sinh^2\theta_1 d\theta_2^2 + \sinh^2\theta_1 \sum\limits_{i=3}^{d-2} \prod\limits_{j=2}^{i-1} \sin^2\theta_j d\theta_i^2 & k=-1 \\ \sum\limits_{i=1}^{d-2} d\theta_i^2 & k=0 \end{cases}, \tag{5}$$

Considering the field equation (2) with the metric (4), one can obtain the following independent sourceless ($\mathrm{T}_{\mu\nu}^{matt} = 0$) field equations;

$$2rg(r)F_R'' + [rg'(r) + 2(d-2)g(r)] F_R' - [rg''(r) + (d-2)g'(r)] F_R = rF(R), \tag{6}$$

$$[rg'(r) + 2(d-2)g(r)] F_R' - [rg''(r) + (d-2)g'(r)] F_R = rF(R), \tag{7}$$

$$2rg(r)F_R'' + 2[rg'(r) + (d-3)g(r)] F_R' - \frac{2(d-3)}{r}\left(\frac{rg'(r)}{d-3} + g(r) - k\right) F_R = rF(R), \tag{8}$$

corresponding to tt, rr and $\theta_i\theta_i$ ($i = 1, 2, ..., d-2$) components of Eq. (2), respectively. It is notable that the prime and double prime are the first and second derivatives with respect to r , respectively. In this paper, we study black hole solutions with constant Ricci scalar (so $F_R'' = F_R' = 0$), and therefore it is easy to show that the field equations (6)-(8) reduce to

$$(rg''(r) + (d-2)g'(r)) F_R = -rF(R), \tag{9}$$

$$\frac{2(d-3)}{r}\left(\frac{rg'(r)}{d-3} + g(r) - k\right) F_R = -rF(R). \tag{10}$$

Equating the left hand sides of Eqs. (9) and (10), we obtain

$$r^2 g''(r) + (d-4)rg'(r) - 2(d-3)g(r) + 2(d-3)k = 0, \tag{11}$$

with the trivial topological Schwarzschild solution $g(r) = k - \frac{R}{d(d-1)}r^2 - \frac{M}{r^{d-3}}$, where $R = \frac{2d\Lambda}{d-2}$. It is important to note that we are looking for a solution which satisfy both Eqs. (9) and

(10) simultaneously and the mentioned trivial topological Schwarzschild solution is not the solution of them for arbitrary $F(R)$. Inserting the mentioned topological Schwarzschild solution (with nonzero R) in Eqs. (9) and (10), and solve new equations, directly, one can find that the topological Schwarzschild solution in Eqs. (9) and (10) leads to exponential form of $F(R)$ gravity ($F(R) = Ce^{\frac{(d-2)R}{4\Lambda}}$), which is not in the form of $F(R) = R + f(R)$. It is notable that in special case, $F(R) = R$ (with constant R), we encounter with sourceless Einstein gravity in the absence of cosmological constant which leads to $R = 0$ (see Eq. (3) for more detail). Therefore it is straightforward to show that for $F(R) = R$ and so $F_R = 1$, Eqs. (9) and (10) reduce to

$$\begin{aligned} rg''(r) + (d-2)g'(r) &= 0, && (12)\\ \frac{rg'(r)}{d-3} + g(r) - k &= 0. && (13) \end{aligned}$$

which their general solution is $g(r) = k - \frac{M}{r^{d-3}}$.

In this paper, we consider some various class of $F(R) = R+f(R)$ gravity and surprisingly, we find that for special cases of $F(R)$ gravity we can obtain charged black holes in addition to Schwarzschild solutions. It is notable that, in general, charged solution could not satisfy Eqs. (9) and (10) and we should set some free parameters in $F(R)$ models to satisfaction.

Here, we want to obtain the static solutions of Eqs. (2) without any matter field ($\mathrm{T}_{\mu\nu}^{matt} = 0$). We assume that the metric has the same form of Eq. (4). Here, to find the function $g(r)$, one may use any components of Eq. (2) with defined function $F(R)$. In what follows, we consider the some special cases of $F(R)$ gravity and investigate their properties.

III. BLACK HOLE SOLUTIONS OF THE MODIFIED $F(R)$ GRAVITY

1. Case (I): Solutions for $F(R) = R - \frac{\mu^4}{R}$ Model:

One of the initiative $F(R)$ models supposed to explaining the positive acceleration of expanding Universe has $F(R)$ action as $F(R) = R - \frac{\mu^4}{R}$ [32]. In this model, for large values of Ricci scalar, $F(R)$ function tends to $F(R) = R$, so we expect for these values of R the modification become negligible but for small values of Ricci scalar, $F(R)$ action is not the linear one thus for this values of Ricci scalar, gravity is modified.

Looking at equations (4) and (2), one could obviously obtain the metric functions as

follows

$$g(r) = k - \frac{2\Lambda}{(d-1)(d-2)}r^2 - \frac{M}{r^{d-3}}. \tag{14}$$

where we should set $\Lambda = \frac{\pm\mu^2}{2d}\sqrt{d^2-4}$. It is easy to show that in this model the Ricci scalar and the Kretschmann scalar are $R = \pm\sqrt{\frac{d+2}{d-2}}\mu^2$ and $R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} = \frac{2(d+2)\mu^4}{d(d-1)(d-2)} + \frac{(d-1)(d-2)^2(d-3)M^2}{r^{2(d-1)}}$, respectively, and so we encounter with a curvature singularity at $r = 0$. In other word, this solution corresponds to the topological Schwarzschild-adS(dS) black hole solution, when we set $\mu^2 = \frac{-2d\Lambda}{\sqrt{d^2-4}}$, $\Lambda < 0$ ($\mu^2 = \frac{2d\Lambda}{\sqrt{d^2-4}}$, $\Lambda > 0$). In summary, we start with pure $F(R)$ gravity in the absence of cosmological constant, but this solution (with $\mu^2 = \frac{\mp 2d\Lambda}{\sqrt{d^2-4}}$) is matched to the Einstein gravity in the presence of cosmological constant, exactly. So, one may think that the cosmological constant could emerge from $F(R)$ gravity. It is notable that in this model we cannot obtain charged solution.

Because for large values of R the modification is negligible, this model can not explain the inflation but there is several viable models unifying inflation and late time acceleration. Also, in this model and for positive cosmological constant (dS solutions), $\frac{d^2F}{dR^2} \equiv F_{RR} = -\frac{(d-2)^3\mu^4}{4d^3\Lambda^3} < 0$, and so one can deduce the presented dS model suffer the Dolgov-Kawasaki instability [25]. It is notable that one can remove instability of dS solutions by adding R^2 term in the mentioned model [33]. In addition, this model is not consistent with solar system tests, exactly [32, 34].

2. Case (II): Solutions for $F(R) = R + \kappa R^n$ Model:

Investigation of $F(R)$ gravity models show that there are some known examples of viable $F(R)$ gravity that exhibit no problems in the weak gravity regime [35]. But for the strong gravity regime, these modes have a serious drawback such as singularity problem. In order to resolve the singularity problem arising in the strong gravity regime, Kobayashi and Maeda [19] have considered a higher curvature correction proportional to R^n where $n > 1$. The field equations, given in Eq. (2) with the metric (4), in this case simply provide

$$\begin{aligned} g(r) &= k - \frac{2\Lambda}{(d-1)(d-2)}r^2 - \frac{M}{r^{d-3}}, \\ \kappa &= \frac{(d-2)^n(2d\Lambda)^{1-n}}{(2n-d)}, n \neq \frac{d}{2}. \end{aligned} \tag{15}$$

Here, we should mention that in this model we cannot obtain charged solution. Straightforward calculations show that our information about the Ricci and the Kretschmann scalars

are

$$R = \frac{2d\Lambda}{(d-2)}, \tag{16}$$

$$\lim_{r\longrightarrow\infty} R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} = \frac{8d\Lambda^2}{(d-2)^2(d-1)}, \tag{17}$$

$$\lim_{r\longrightarrow 0} R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} \propto \lim_{r\longrightarrow 0} r^{-2(d-1)} = \infty. \tag{18}$$

These information help us to conclude that there is a curvature singularity at $r=0$, and this solution is correspond with the topological Schwarzschild black hole in the presence of cosmological constant. In other word, one can extract the cosmological constant from $F(R) = R + \kappa R^n$ gravity by suitable coefficient κ. For $n=2$, the solution reduces to topological Schwarzschild black hole for vanishing Λ. It is simple to calculate F_{RR}

$$F_{RR} = \frac{n(n-1)(d-2)^2}{2d\Lambda(2n-d)}, \tag{19}$$

and therefore one can find that the adS (dS) solutions follow Dolgov-Kawasaki stability for $n < d/2$ ($n > d/2$).

3. Case (III): Solutions for $F(R) = R - \lambda \exp(-\xi R)$ Model:

It is notable that viable modifications in gravity should pass all sorts of tests from the large scale structure of the universe to galaxy and cluster dynamics to the solar system tests. One of the outstanding question in $F(R)$ gravity is whether this theory is consistent with solar system tests or not. When the correction term of Einstein gravity is exponential form (see for e.g. [4, 36]), one can prove that in this model, there is no conflict with solar system tests and also it satisfy high curvature condition [37]. In addition, the solutions of this model is virtually indistinguishable from that in general relativity, up to a change in the Newton's constant [37]. Considering the presented $F(R)$ model, with Eqs. (4) and (2), we clearly deduce at least two type solutions:

As one can check easily, the first solution is corresponding to the Schwarzschild (a)dS in the following form

$$g(r) = k - \frac{2\Lambda}{(d-1)(d-2)}r^2 - \frac{M}{r^{d-3}}. \tag{20}$$

when we set $\lambda = \frac{2d\Lambda e^{\xi R}}{d+2\xi R}$ with Ricci scalar R which is calculated in Eq. (16) and free parameter

ξ. Second solution is the same as topological charged solution

$$g(r) = k - \frac{2\Lambda}{(d-1)(d-2)} r^2 - \frac{M}{r^{d-3}} + \frac{Q^2}{r^{d-2}}. \tag{21}$$

where we should set some parameters. Considering charged solution, Eq. (21) with $F(R) = R - \lambda \exp(-\xi R)$, one can show that Eqs. (6)-(8) reduce to

$$\left(1 + \frac{\lambda\xi}{e^{\xi R}}\right)(d-2)Q^2 - \left[\frac{\lambda}{e^{\xi R}} + \frac{2R}{d}\left(\frac{\lambda\xi}{e^{\xi R}} - \frac{d-2}{2}\right)\right] r^d = 0. \tag{22}$$

In order to vanish Eq. (22) we should consider the following equations

$$\begin{aligned} &1 + \frac{\lambda\xi}{e^{\xi R}} = 0, \\ &\frac{\lambda}{e^{\xi R}} + \frac{2R}{d}\left(\frac{\lambda\xi}{e^{\xi R}} - \frac{d-2}{2}\right) = 0. \end{aligned} \tag{23}$$

with the solutions $\lambda = Re^{-1}$ and $\xi = -1/R$. In other words, setting $\lambda = Re^{-1}$ and $\xi = -1/R$, one can show that the charged solution (21) satisfies field equations of the mentioned $F(R)$ gravity. We should note that the presented uncharged black holes (20) is the same as solutions which obtained in Ref. [38] for $\alpha = 0$. Although for $g_{tt} = g_{rr}^{-1}$ in Ref. [38], the metric function has a charged term $\frac{C_1}{r^2}$ in analogy with our charged solution ($d = 4$), but these solutions are completely different. They have different geometry and constant curvature, R, with various asymptotic behavior.

In addition, it is notable that the metric function $g(r)$, presented here, differ from the standard higher-dimensional Reissner–Nordström solutions since the electric charge term in the metric function (21) is proportional to $r^{-(d-2)}$ but in the standard higher-dimensional charged black hole solutions is proportional to $r^{-2(d-3)}$. In order to interpret the charge term, one may think about the scalar-tensor representation of $F(R)$ gravity. In order to find a scalar-tensor representation of $F(R)$ theories, one can use conformal transformations [31]. Considering the conformal transformations on the metric function, we find that there no new interpretation about new metric function unless its scale.

On other hand, comparing the uncharged solution of $F(R) = R + f(R)$ gravity (Eq. (20)) with the solution of Einstein gravity in the presence of cosmological constant shows that $f(R)$ has the role of cosmological constant. As we will see, we compare the charged solution of $F(R) = R + f(R)$ gravity (Eq. (21)) with the solution of Einstein gravity coupled to the conformally invariant Maxwell field and state that $f(R)$ gravity has the role of electromagnetic field in addition to the cosmological constant.

By calculating the Ricci and the Kretschmann scalars, we find

$$R = \frac{2d\Lambda}{d-2}, \tag{24}$$

$$\lim_{r\longrightarrow\infty} R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} = \frac{8d\Lambda^2}{(d-2)^2(d-1)}, \tag{25}$$

$$\lim_{r\longrightarrow 0} R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} \propto \left\{ \begin{array}{ll} \lim_{r\longrightarrow 0} r^{-2(d-1)} & \text{First solution Eq. (20)} \\ \lim_{r\longrightarrow 0} r^{-2d} & \text{Second solution Eq. (21)} \end{array} \right. = \infty. \tag{26}$$

in which we can deduce that there is a essential singularity at $r = 0$. It is notable that the second derivative of the $F(R)$ function for this specific model is

$$F_{RR} = -\frac{d-2}{2d\Lambda}, \tag{27}$$

and so we find that adS solutions are stable in arbitrary dimensions.

4. Case (IV): Solutions for $F(R) = R - \lambda\exp(-\xi R) + \kappa R^2$ Model:

However, for $F(R) = R - \lambda\exp(-\xi R)$, the solutions are unstable, one can add another small correction to make it stable in the solar system. We choose R^2 correction, which was firstly proposed in Ref. [36, 39] to explain inflation. Thus the total model is $F(R) = R - \lambda\exp(-\xi R) + \kappa R^2$, where κ is constant. By adjust the parameters, this new form of $F(R)$ can satisfy the high curvature condition, early universe inflation, stability and the late time acceleration [37]. In the model, we can obtain at least two solutions. First solution is uncharged and is like Eq. (20) with the same condition on λ. Second solution is corresponding with the topological Einstein-nonlinear Maxwell solution in the presence of cosmological constant

$$g(r) = k - \frac{2\Lambda}{(d-1)(d-2)}r^2 - \frac{M}{r^{d-3}} + \frac{Q^2}{r^{d-2}}. \tag{28}$$

which we should fix some parameters like λ and κ. Inserting this solution, Eq. (28), in the field equations (6)-(8) with the mentioned model, one can conclude that the field equations reduce to

$$\begin{aligned} &\left[1 + 2R\kappa + \xi\lambda e^{-\xi R}\right] d(d-2)Q^2 - \\ &\left[(2R\xi + d)\,\lambda e^{-\xi R} + (d-4)\kappa R^2 - (d-2)R\right] r^d = 0 \end{aligned} \tag{29}$$

which we should adjust $\lambda = \frac{Re^{\xi R}}{2+\xi R}$, $\kappa = -\frac{1+\xi R}{R(2+\xi R)}$ for satisfaction, and ξ is free parameter. These solutions are very close to previous solutions and there is a curvature singularity at $r = 0$. It is matter of calculation to show that

$$F_{RR} = -\frac{(d-2+d\xi\Lambda)^2 + \xi^2 d^2 \Lambda^2}{2d\Lambda(d-2+d\xi\Lambda)}, \tag{30}$$

and therefore one can set free parameter ξ to obtain positive value for F_{RR} for arbitrary value of cosmological constant.

5. *Case (V): Solutions for $F(R) = R - \lambda\exp(-\xi R) + \kappa R^n + \eta\ln(R)$ Model:*

In this subsection, we generalize previous types of $F(R)$ gravity and consider a combination of the mentioned corrections in previous subsections. Let us take $F(R)$ in the following 5-parametric form

$$F(R) = R - \lambda\exp(-\xi R) + \kappa R^n + \eta\ln(R). \tag{31}$$

The solutions of Eqs. (4) and (2), with this choice of $F(R)$, can be written as two kind of charged and uncharged solutions like previous section, but with different conditions on parameters. If we consider topological Schwarzschild

$$g(r) = k - \frac{2\Lambda}{(d-1)(d-2)}r^2 - \frac{M}{r^{d-3}}. \tag{32}$$

in the field equations (6)-(8) with the mentioned model, we obtain

$$\eta d\ln R - (d+2\xi R)\,\lambda e^{-\xi R} + (d-2)R + (2n-d)\,\kappa R^n - 2\eta = 0. \tag{33}$$

which it help us to find that in the presence of cosmological constant we should set

$$\lambda = \frac{\eta d\ln R + (d-2)R + (2n-d)\,\kappa R^n - 2\eta}{(d+2\xi R)\,e^{-\xi R}}, \tag{34}$$

and ξ, κ, η are free parameters. In order to study Dolgov-Kawasaki stability, we should calculate F_{RR}

$$\begin{aligned} F_{RR} \;=\; & \frac{\xi^2 e^{\frac{\xi[(d-2)R-2d\Lambda]}{d-2}}}{d+2\xi R}\left[(d-2n)R^n\kappa - (d-2)R - (d\ln R - 2)\eta\right] + \\ & R^{-2}\left[n(n-1)R^n\kappa - \eta\right]. \end{aligned} \tag{35}$$

It is not simple to present a condition for stability, but as one can see in Fig. 1, we can set

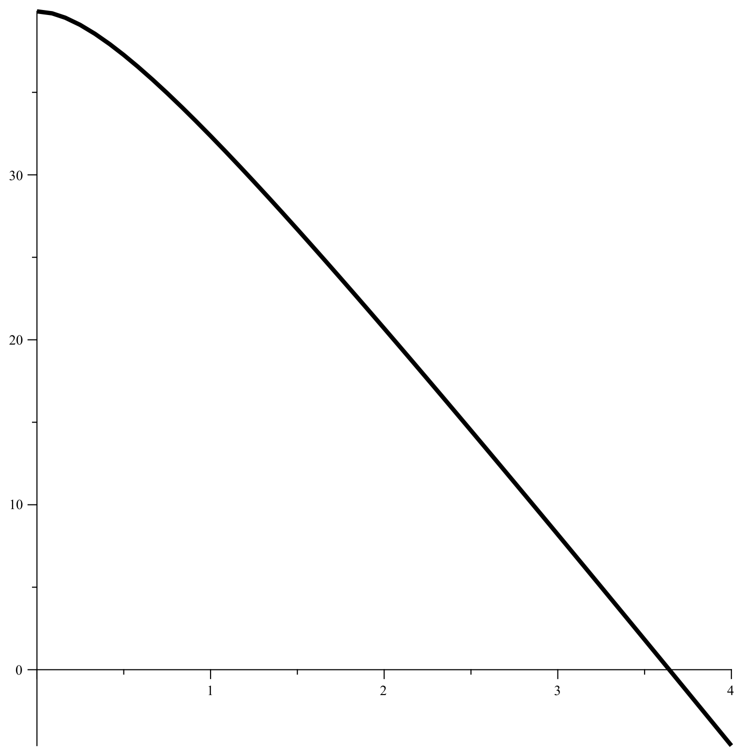

FIG. 1: F_{RR} (Eq. 35) versus ξ for $d = 5$, $\Lambda = 1$, $\kappa = 2$, $n = 3$ and $\eta = 1$.

free parameters to obtain positive F_{RR}.

When we consider topological charged solution with nonzero Λ,

$$g(r) = k - \frac{2\Lambda}{(d-1)(d-2)} r^2 - \frac{M}{r^{d-3}} + \frac{Q^2}{r^{d-2}}. \tag{36}$$

the field equations (6)-(8) reduce to

$$\begin{aligned} &\left[\left(1 + \xi\lambda e^{-\xi R}\right) R + n\kappa R^n + \eta\right] d(d-2)Q^2 - \\ &\left[(2\xi R + d)\, \lambda e^{-\xi R} - \eta d \ln R - (d-2)R + (2n-d)\,\kappa R^n + 2\eta\right] R r^d = 0 \end{aligned} \tag{37}$$

In order to satisfy Eq. (37) with nonzero Q and Λ, the coefficients of Λr^d and Q^2 should set to zero, separately, to obtain

$$\lambda = \frac{R + \kappa R^n - (R + n\kappa R^n) \ln R}{(1 + \xi R \ln R)\, e^{-\xi R}}, \tag{38}$$

$$\eta = -\frac{(1 + \xi R)\, R + (n + \xi R)\, \kappa R^n}{1 + \xi R \ln R}. \tag{39}$$

Considering both of Eqs. (32) and (36), one can show that the asymptotic behavior of these solutions near the origin ($r = 0$) and for large value of r ($r \longrightarrow \infty$) is the same as

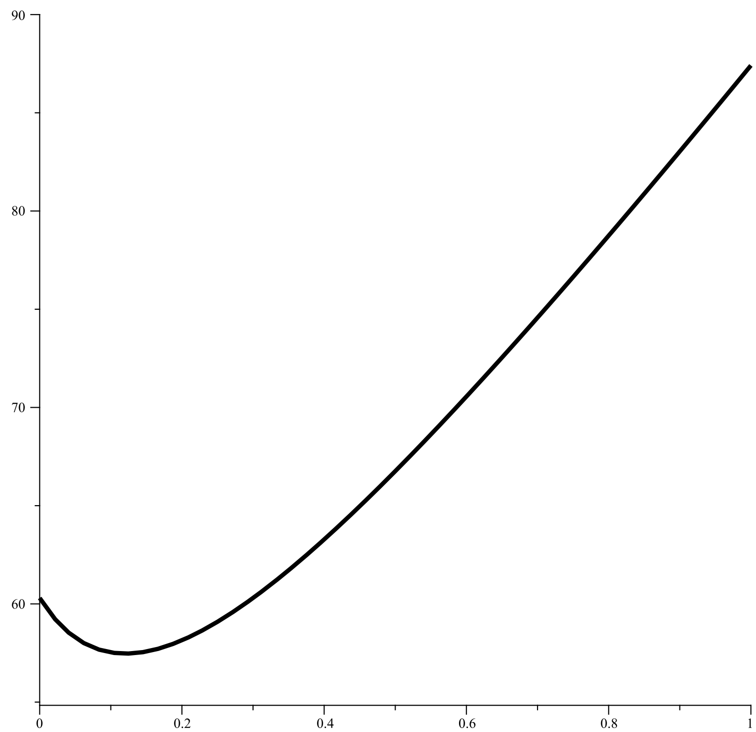

FIG. 2: F_{RR} (Eq. 40) versus ξ for $d = 5$, $\Lambda = 1$, $\kappa = 2$ and $n = 3$.

presented expressions in Eqs. (24)-(26). Here we want to discuss about Dolgov-Kawasaki stability. Using Eqs. (38) and (39) in second derivative of Eq. (31), yealds

$$\begin{aligned}
F_{RR} &= \frac{(d-2)\left(\Psi_1+\Psi_2+\Psi_3\right)}{(2\xi d\Lambda \ln R + d - 2)}, \\
\Psi_1 &= \frac{2d\Lambda(\ln R - 1)}{(d-2)}\left(\frac{(d-2)R^n\kappa(n\ln R - 1)}{2d\Lambda(\ln R - 1)} + 1\right)\xi^2, \\
\Psi_2 &= \left(\frac{(d-2)R^n\kappa\left[n(n-1)\ln R + 1\right]}{2d\Lambda} + 1\right)\xi, \\
\Psi_3 &= \frac{(d-2)}{2d\Lambda}\left(\frac{(d-2)R^n\kappa n^2}{2d\Lambda} + 1\right).
\end{aligned} \tag{40}$$

We plot Eq. (40) versus ξ in Fig. 2, for some fixed free parameters and find that F_{RR} is positive for some values of ξ.

6. *Case (VI): Solutions for the Starobinsky and the Generalized Starobinsky Models:*

One of the $F(R)$ models that pass all the observational and theoretical constraints is the Starobinsky model [39, 40]. This model is proposed which produce viable cosmology

different from the ΛCDM one at recent times and satisfy cosmological, Solar system and laboratory tests. In this model the $F(R)$ function define as

$$F(R) = R + \lambda R_0 \left(\left(1 + \frac{R^2}{R_0^2} \right)^{-n} - 1 \right), \tag{41}$$

where λ, R_0 and n are constants.

Looking at equations (4) and (2), one could obviously obtain the metric functions as follows

$$g(r) = k - \frac{2\Lambda}{(d-1)(d-2)} r^2 - \frac{M}{r^{d-3}}, \tag{42}$$

with a constrain on the free parameters as

$$\begin{aligned} &\frac{\lambda R_0}{dV^n} \left[dR_0^2 + (d+4n) R^2 \right] - (\lambda R_0 - 2\Lambda) V R_0^2 = 0, \\ &V = \frac{R_0^2 + R^2}{R_0^2}. \end{aligned} \tag{43}$$

Considering the Eq. (43), it is easy to show that all field equations are satisfied when $\lambda = -2\Lambda V R_0 \left(\left[R_0^2 + R^2 \left(1 + \frac{4n}{d} \right) \right] V^{-n} - V R_0^2 \right)^{-1}$. Here we conclude that this the solution of this model is corresponding to the Schwarzchild (a)dS solution in Einstein gravity. Straightforward calculations show that in this case we cannot obtain charged solution. It is notable that for uncharged solution we obtain

$$F_{RR} = \frac{4nd\Lambda \left[1 - \frac{2(n+1)R^2}{VR_0^2} \right]}{VR_0^2 d(1 - V^n) + 4nR^2}, \tag{44}$$

where, as one can follow in Fig. 3, we obtain stable (a)dS solutions for suitable R_0.

In order to obtain the topological charged solution, we generalize this model with adding an extra term to Starobinsky model. We define the generalized Starobinsky model in the following form

$$F(R) = R + \lambda R_0 \left(\left(1 + \frac{R^2}{R_0^2} \right)^{-n} - 1 \right) + \kappa R^n. \tag{45}$$

The basic motivation for this extra term is that for $R_0 << R$, the Starobinsky model reduce to Einstein gravity with R^2 correction and in the generalized Starobinsky model, we generalize the power of correction term to arbitrary factor n. The solution of Eqs. (4) and (2), in the generalized Starobinsky model is as follows

$$g(r) = k - \frac{2\Lambda}{(d-1)(d-2)} r^2 - \frac{M}{r^{d-3}} + \frac{Q^2}{r^{d-2}}, \tag{46}$$

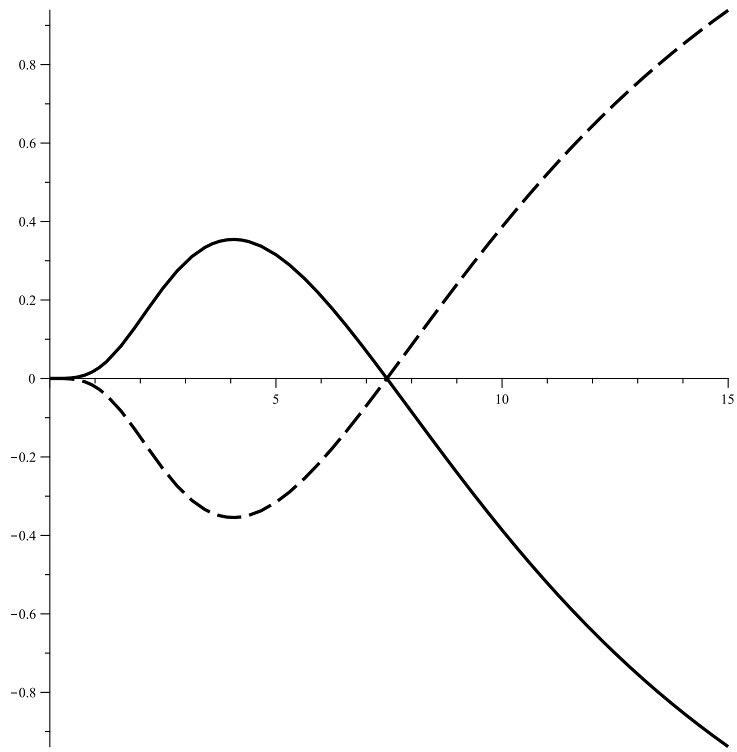

FIG. 3: F_{RR} (Eq. 44) versus R_0 for $d=5$, $n=2$, and $\Lambda=1$ (solid line) and $\Lambda=-1$ (dashed line).

with two constrain on the free parameters which appear in the following equation

$$\left(\kappa R^n V R_0^2\left(2n-d\right)-d\lambda R_0 V^{-n}\left[R_0^2+R^2\left(1+\frac{4n}{d}\right)\right]+VR_0^2\left(d\lambda R_0-2d\Lambda\right)\right)Rr^d+$$
$$\left[-VR_0^2\left(R+n\kappa\left(R\right)^n\right)+2R^2n\lambda R_0V^{-n}\right]d(d-2)Q^2=0. \tag{47}$$

which for nonzero Q and Λ, we obtain

$$\lambda = -\frac{R\left(n-1\right)}{n\left(\left[R_0^2+3R^2\right]V^{-(2n+1)/2}-R_0\right)}, \tag{48}$$
$$\kappa = \frac{R_0-\left[R^2\left(2n+1\right)+R_0^2\right]V^{-(2n+1)/2}}{nR^{n-1}\left[\left(R_0^2+3R^2\right)V^{-(2n+1)/2}-R_0\right]}. \tag{49}$$

Considering both of Eqs. (42) and (46), one can show that the asymptotic behavior of these solutions near the origin ($r=0$) and for large value of r ($r\longrightarrow\infty$) is the same as presented expressions in Eqs. (24)-(26). Finally we calculate second derivative of $F(R)$ gravity in generalized Starobinsky model to obtain

$$F_{RR} = \frac{-2n\left(n-1\right)R\left[2(n+1)R^2-VR_0^2\right]}{nR_0^3V^{n+2}\left[\left(R_0^2+3R^2\right)V^{-(2n+1)/2}-R_0\right]}+$$
$$\frac{R_0-\left[R^2\left(2n+1\right)+R_0^2\right]V^{-(2n+1)/2}}{\frac{R}{n-1}\left[\left(R_0^2+3R^2\right)V^{-(2n+1)/2}-R_0\right]}, \tag{50}$$

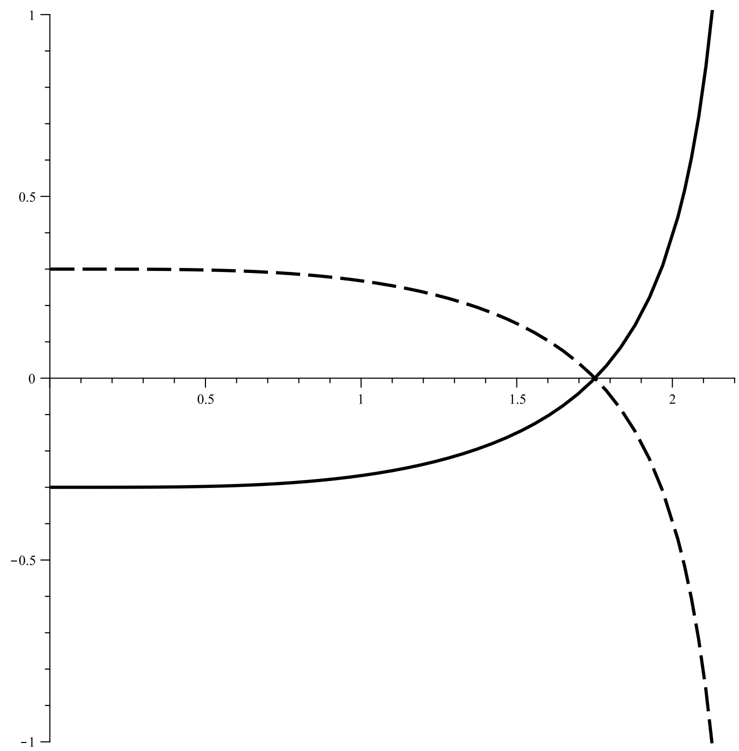

FIG. 4: F_{RR} (Eq. 50) versus R_0 for $d = 5$, $n = 2$, and $\Lambda = 1$ (solid line) and $\Lambda = -1$ (dashed line).

where we plot it in Fig. 4. This figure shows that we can obtain stable solution for special values of R_0 for positive and negative cosmological constant.

IV. BLACK HOLE SOLUTIONS IN EINSTEIN GRAVITY WITH CONFORMALLY INVARIANT MAXWELL FIELD

In this section, we consider the d-dimensional Einstein-Λ Gravity with Conformally Invariant Maxwell Field (EΛCIM). In other word, the electromagnetic field equation be invariant under conformal transformation ($g_{\mu\nu} \longrightarrow \Omega^2 g_{\mu\nu}$ and $A_\mu \longrightarrow A_\mu$). For more investigations and motivations one can see Ref. [41]. The action of EΛCIM is

$$\mathcal{I}_{E\Lambda CIM} = -\frac{1}{16\pi} \int\limits_{\partial M} d^{n+1}x\sqrt{-g}\left[R - 2\Lambda - \alpha(F_{\alpha\beta}F^{\alpha\beta})^{d/4}\right], \tag{51}$$

where α is a coupling constant. In order to ensure a physical interpretation of our future solutions, we fix $\alpha = (-1)^{1-d/4}$ and so the energy density, i.e. the $T_{\widehat{0}\widehat{0}}$ component of the energy-momentum tensor in the orthonormal frame, is positive. Varying the action (51) with respect to the metric tensor $g_{\mu\nu}$ and the electromagnetic field A_μ, the equations of

gravitational and electromagnetic fields may be obtained as

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{\alpha}{2}\left[dF_{\mu\rho}F_{\nu}^{\rho}(F_{\alpha\beta}F^{\alpha\beta})^{d/4-1} - g_{\mu\nu}(F_{\alpha\beta}F^{\alpha\beta})^{d/4}\right], \tag{52}$$

$$\partial_{\mu}\left[\sqrt{-g}F^{\mu\nu}(F_{\alpha\beta}F^{\alpha\beta})^{d/4-1}\right] = 0. \tag{53}$$

The Maxwell equation (53) with metric (4) can be integrated immediately to give

$$F_{tr} = \frac{q}{r^2}, \tag{54}$$

where q, an integration constant and it is proportional to the electric charge of the spacetime. Here, we calculate the electric charge per unit volume of the black hole by finding the flux of the electromagnetic field at infinity, obtaining

$$\mathcal{Q} = \frac{2^{d/4-1}dq^{d/2-1}}{16\pi}, \tag{55}$$

which confirm that q is related to the electrical charge of the spacetime $\mathcal{Q}$.

Inserting the Maxwell fields (54) and the metric (4) in the field equation (52), one can show that these equations have the following solutions

$$g(r) = k - \frac{2\Lambda}{(d-1)(d-2)}r^2 - \frac{M}{r^{d-3}} + \frac{2^{d/4-1}q^{d/2}}{r^{d-2}}, \tag{56}$$

where m is the integration constant which is related to mass parameter and when we set $d = 4$, Reissner-Nordström solution is recovered.

Comparing Eq. (56) with Eq. (21) (and let $Q^2 = 2^{d/4-1}q^{d/2}$), show that there is a correspondence between the EΛCIM solutions and the solutions of $F(R)$ gravity without matter field. Considering Eq. (55), one can find

$$\mathcal{Q} = \frac{2^{\frac{d-4}{2d}}dQ^{\frac{2(d-2)}{d}}}{16\pi}, \tag{57}$$

where in four dimension it reduce to total electric charge of Reissner-Nordström spacetime ($\mathcal{Q} = \frac{q}{4\pi} = \frac{Q}{4\pi}$).

V. CONCLUSIONS

In summary we investigated some different well-known types of $F(R)$ theory as a correction to Einstein gravity with constant Ricci scalar metric in higher dimensions. The

basic motivation arise from Einstein dream which is creating a geometrical unified theory of physics. We found two kind of charged and uncharged black hole solutions with constant Ricci scalar and different topology in horizon and investigated some geometrical properties such as singularity and asymptotic behavior.

In this paper, we considered some various class of $F(R) = R + f(R)$ gravity and surprisingly, we found that for special cases of $F(R)$ gravity we can obtain charged black holes in addition to Schwarzschild solutions. It is notable that we started with pure $F(R)$ gravity without matter field and cosmological constant, but we found that, by fixing some of the free parameters, presented charged and uncharged solutions are corresponding with the topological nonlinear charged and the Schwarzschild black holes in the presence of cosmological constant. In other word, it is not necessary to insert (by hand) the cosmological constant in the field equations, and by setting some of free parameters in $F(R)$ gravity, one may obtain (a)dS solution from pure geometry. In addition, one may think about charged solution in $F(R)$ gravity, without considering the (nonlinear-) Maxwell stress-energy tensor.

Also, we calculated Kreschmann scalar to show that there is a curvature singularity at $r = 0$. It means these solutions are interpreted as topological (un)charged black holes. It is notable that the Kreschmann scalar of charged solutions diverges as $r \to 0$, but with a rate slower than that of uncharged solutions. In addition, for any kind of presented $F(R)$ models, we analyze Dolgov-Kawasaki stability and show that one can find stable solutions provided the parameters of the $F(R)$ gravity are chosen suitably.

Finally, it is also be desirable to study the cosmological view, the causal structure and dynamical stability as well as thermodynamical properties of the black hole solutions derived here. Generalization of these solutions to non-constant Ricci scalar with nontrivial topology of horizons as well as obtaining a special transformation to find a corresponding between $F(R)$ gravity and Einstein-nonlinear Maxwell gravity, remain to be carried out in the future.

Acknowledgments

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[1] S. Perlmutter et al., Astrophys. J. 517, 565 (1999);
A. G. Riess et al., Astron. J. 116, 1009 (1998);
A. G. Riess et al., Astrophys. J. 607, 665 (2004);
A. Grant et al, Astrophys. J. 560, 49 (2001);
S. Perlmutter, M. S. Turner and M. White, Phys. Rev. Lett. 83, 670 (1999);
C. L. Bennett et al, Astrophys. J. Suppl. 148, 1 (2003);
G. Hinshaw et al, Astrophys. J. Suppl. 148, 135 (2003).

[2] I. L. Buchbinder, S. D. Odintsov and I. L. Shapiro, "Effective Action in Quantum Gravity" (Bristol: IOP Publishing) (1992);
M. Gasperini and G. Veneziano, Phys. Lett. B 277, 256 (1992);
S. Nojiri and S. D. Odintsov, Phys. Lett. B 576, 5 (2003);
D. V. Vassilevich, Phys. Rep. 388, 279 (2003).

[3] M. Akbar and R. G. Cai, Phys. Lett. B 635, 7 (2006);
M. Akbar and R. G. Cai, Phys. Lett. B 648, 243 (2007);
J. C. C. de Souza and V. Faraoni, Class. Quant. Grav. 24, 3637 (2007);
K. Atazadeh and H. R. Sepangi, Int. J. Mod. Phys. D 16, 687 (2007);
K. Atazadeh, M. Farhoudi and H. R. Sepangi, Phys. Lett. B 660, 275 (2008);
K. Bamba and S. D. Odintsov, JCAP 0804, 024 (2008);
C. Corda, Astropart. Phys. 34, 587 (2011);
C. Corda, Europhys. Lett. 86, 20004 (2009);
C. Corda, Gen. Rel. Grav. 40, 2201 (2008);
C. Corda, Mod. Phys. Lett. A 23, 109 (2008).

[4] G. Cognola, E. Elizalde, S. Nojiri, S.D. Odintsov, L. Sebastiani and S. Zerbini, Phys. Rev. D 77, 046009 (2008).

[5] T. P. Sotiriou and V. Faraoni, Rev. Mod. Phys. 82, 451 (2010);

S. Nojiri and S.D. Odintsov, [arXiv:0601213];

S. Nojiri and S.D. Odintsov, [arXiv:1011.0544];

V. Faraoni, [arXiv:0810.2602];

N. Straumann, [arXiv:0809.5148].

[6] S. H. Hendi, Phys. Lett. B 690, 220 (2010).

[7] M. Halilsoy, T. Tahamtan and S. Habib Mazharimousavi, [arXiv:1110.0094]

[8] G. Cognola, E. Elizalde, S. Nojiri, S. D. Odintsov and S. Zerbini, Phys. Rev. D 73, 084007 (2006).

[9] S. Nojiri and S. D. Odintsov, Phys. Rev. D 74, 086005 (2006);

S. Nojiri and S. D. Odintsov, J. Phys. Conf. Ser. 66, 012005 (2007);

S. Capozziello, S. Nojiri, S. D. Odintsov and A. Troisi, Phys. Lett. B 639, 135 (2006);

S. Nojiri and S. D. Odintsov, Phys. Rev. D 78, 046006 (2008).

[10] A. A. Starobinsky, Phys. Lett. B 91, 99 (1980).

[11] K. Bamba and S. D. Odintsov, JCAP 0804, 024 (2008).

[12] S. M. Carroll, V. Duvvuri, M. Trodden and M. S. Turner, Phys. Rev. D 70, 043528 (2004).

[13] A. de la Cruz-Dombriz and A. Dobado, Phys. Rev. D 74, 087501 (2006).

[14] S. Fay, R. Tavakol and S. Tsujikawa, Phys. Rev. D 75, 063509 (2007).

[15] M. C. B. Abdalla, S. Nojiri and S. D. Odintsov, Class. Quantum Gravit. 22, L35 (2005).

[16] F. Briscese, E. Elizalde, S. Nojiri and S. D. Odintsov, Phys. Lett. B 646, 105 (2007).

[17] S. Nojiri and S. D. Odintsov, Phys. Rev. D 78, 046006 (2008).

[18] K. Bamba, S. Nojiri and S. D. Odintsov, JCAP 0810, 045 (2008).

[19] T. Kobayashi and K. Maeda, Phys. Rev. D 79, 024009 (2009).

[20] S. Capozziello, V.F. Cardone, A. Troisi, JCAP 0608, 001 (2006);

S. Capozziello, V.F. Cardone, A. Troisi, Mon. Not. Roy. Astron. Soc. 375, 1423 (2007);

C. Frgerio Martins and P. Salucci, Mon. Not. Roy. Astron. Soc. 381, 1103 (2007).

[21] C. Corda, Astropart. Phys. 30, 209 (2008);

C. Corda, Int. J. Mod. Phys. A 23, 1521 (2008);

C. Corda, Int. J. Mod. Phys. D 18, 2275 (2009);

C. Corda, Eur. Phys. J. C 65, 257 (2010);

C. Corda, Gen. Rel. Grav. 42, 1323 (2010); Erratum-ibid. 42, 1335 (2010);

C. Corda, Int. J. Mod. Phys. D 19, 2095 (2010);

C. Corda, Astropart. Phys. 34, 412 (2011).

[22] S. Capozziello, A. Troisi, Phys. Rev. D 72, 044022 (2005).

[23] S. Capozziello, A. Stabile and A. Troisi, Phys. Rev. D 78, 104019 (2007).

[24] R. P. Woodard, Lect. Notes Phys. 720, 403 (2007).

[25] A. D. Dolgov and M. Kawasaki, Phys. Lett. B 573, 1 (2003);
V. Faraoni, Phys. Rev. D 74, 104017 (2006);
I. Sawicki and W. Hu, Phys. Rev. D 75, 127502 (2007).

[26] B. Whitt, Phys. Lett. B 145, 176 (1984);
J. D. Barrow and S. Cotsakis, Phys. Lett. B 214, 515 (1988);
K. I. Maeda, Phys. Rev. D 39, 3159 (1989);
S. Gottlöber, H.-J. Schmidt and A. A. Starobinsky, Class. Quant. Grav. 7, 893 (1990).

[27] S. Capozziello, A. Stabile and A. Troisi, Class. Quantum Grav. 24, 2153 (2007);
S. Capozziello, A. Stabile and A. Troisi, Class. Quantum Grav. 25, 085004 (2008);
T. Multamaki and I. Vilja, Phys. Rev. D 74, 064022 (2006);
A. Azadi, D. Momeni and M. Nouri-Zonoz, Phys. Lett. B 670, 210 (2008);
D. Momeni and H. Gholizade, Int. J. Mod. Phys. D 18, 1719 (2009) ;
M. Sharif and M. Farasat Shamir, Mod. Phys. Lett. A 25, 1281 (2010) ;
A. M. Nzioki, S. Carloni, R. Goswami and P. K.S. Dunsby, Phys. Rev. D 81, 084028 (2010).

[28] L. Hollenstein and F. S. N. Lobo, Phys. Rev. D 78, 124007 (2008).

[29] A. de la Cruz-Dombriz, A. Dobado and A. L. Maroto, Phys. Rev. D 80, 124011 (2009).

[30] T. Moon, Y. S. Myunga and E. J. Sona, [arXiv:1101.1153].

[31] S. Capozziello, S. Nojiri and S. D. Odintsov, Phys. Lett. B 634, 93 (2006);
T. Faulkner, M. Tegmark, E. F. Bunn and Y. Mao, [arXiv:astro-ph/0612569];
V. Faraoni, [arXiv:gr-qc/0703044];
Y. Bisabr, [arXiv:0808.2133];
M. Capone and M. L. Ruggiero, [arXiv:0910.0434].

[32] S. M. Carroll, et al., Phys. Rev. D 70, 043528 (2004).

[33] S. Nojiri and S. D. Odintsov, Phys. Rev. D 68, 123512 (2003).

[34] T. Chiba, Phys. Lett. B 575, 1 (2003).

[35] S. Tsujikawa, Phys. Rev. D 77, 023507 (2008);
S. Capozziello and S. Tsujikawa, Phys. Rev. D 77, 107501 (2008);

P. Brax, C. van de Bruck, A. C. Davis and D. J. Shaw, Phys. Rev. D 78, 104021 (2008).

[36] S. Nojiri and S. D. Odintsov, [arXiv:1008.4275];

E. Elizalde, S. Nojiri, S. D. Odintsov, L. Sebastiani and S. Zerbini, [arXiv:1012.2280].

[37] P. Zhang, Phys. Rev. D 76, 024007 (2007).

[38] L. Sebastiani and S. Zerbini, Eur. Phys. J. C 71, 1591 (2011).

[39] A. A. Starobinsky and H. J. Schmidt, Class. Quantum Grav. 4, 695 (1987).

[40] A. A. Starobinsky, J. Exp. Theor. Phys. Lett. 86, 157 (2007);

V. Faraoni, [arXiv:0810.2602].

[41] M. Hassaine and C. Martinez, Phys. Rev. D 75 027502 (2007);

S. H. Hendi and H. R. Rastegar-Sedehi, Gen. Rel. Grav. 41, 1355 (2009);

S. H. Hendi, Phys. Lett. B 677, 123 (2009).